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Soft Gravitons Screen Couplings in de Sitter Space

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Abstract

The scale invariance of the quantum fluctuations in de Sitter space leads to the appearance of de Sitter symmetry breaking infra-red logarithms in the graviton propagator. We investigate physical effects of soft gravitons on the local dynamics of matter fields well inside the cosmological horizon. We show that the IR logarithms do not spoil Lorentz invariance in scalar and Dirac field theory. The leading IR logarithms can be absorbed by a time dependent wave function renormalization factor in the both cases. In the interacting field theory with $\lambda\phi^4$ and Yukawa interaction, we find that the couplings become time dependent with definite scaling exponents. We argue that the relative scaling exponents of the couplings are gauge invariant and physical as we can use the evolution of a coupling as a physical time.

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1 Introduction

In de Sitter (dS) space, the degrees of freedom outside the cosmological horizon increase with cosmic evolution. This increase leads to the de Sitter symmetry breaking term in the propagator of a massless and minimally coupled scalar field. It is a direct consequence of the scale invariant fluctuation spectrum [1, 2, 3]. So in some field theoretic models in dS space, physical quantities may become time dependent through the propagator. The gravitational field on dS background is a candidate which induces such infra-red (IR) effects. It is because the gravitational field is massless and contains minimally coupled modes [4].

In order to investigate interacting field theories in dS space, we need to employ the Schwinger-Keldysh perturbation theory [5, 6]. The IR effects at each order manifest as the polynomials in the logarithm of the scale factor of the universe: $\log a(\tau)$ [7]. In an interacting scalar field theory with polynomial interactions such as $\lambda\varphi^4$ theory [8], this IR logarithms in the propagators give rise to powers of IR logarithms in the amplitudes. Let us consider the expectation value of the energy-momentum tensor for example. The leading power of IR logarithms is given by the number of the propagators of the diagram. They make an effective cosmological constant time dependent and thus break de Sitter symmetry. Therefore we need to sum these leading IR logarithms to understand the long term evolution as the effect of IR logarithms becomes large if we wait long enough.

Remarkably a simple physical picture holds in the leading IR log approximation as follows. A scalar field is not completely frozen beyond the horizon scale as it is constantly jolted by the modes coming out of the horizon. A scalar field performs a random walk in the scalar field space (1 dimension) which is consistent with the linear growth of the propagator with respect to cosmic time t at the initial stage. ‡ Eventually it reaches an equilibrium state in a potential well as de Sitter symmetry is restored [9, 10]. We have investigated IR logarithms in the non-linear sigma models which contain massless minimally coupled scalar fields with derivative interactions. In the expectation value of the energy-momentum tensor, we have shown that the leading IR logarithms cancel to all orders [11].

It is important to understand the IR effects in quantum gravity as the propagator of gravitons contains IR logarithms in de Sitter space. The case is very strong here as gravitons exist in our Universe which is of de Sitter type. In this paper we investigate physical effects of soft gravitons on microscopic physics in de Sitter space. We focus on the dynamics of matter fields with sub-horizon momentum scale as it is directly observable. In contrast, the momentum scale of matter fields is taken to be super-horizon scale in [12, 13]. We find that the super-horizon gravitons influence the matter field dynamics inside the cosmological horizon. Our predictions are thus verifiable by direct observations in principle.

In investigating the soft graviton effects to local matter field dynamics, it is a non-trivial question whether they preserve the Lorentz invariance. To verify the Lorentz invariance, we investigate the kinetic terms of matter fields, specifically the relative weight between the time derivative term and the spatial derivative term. In interacting field theories we investigate how the IR effects influence their coupling constants. As specific examples, we adopt $\lambda\phi^4$ and Yukawa theory with dimensionless couplings .

‡We recall here the fractal dimension of random walk is 2.

The organization of this paper is as follows. In Section 2, we quantize gravitational field on dS background. We identify the graviton modes which exhibit IR logarithm. In Section 3, we evaluate the quantum equation of motion with respect to matter fields which are dressed by soft gravitons at the one-loop level. First, we adopt free field theories and investigate whether the Lorentz invariance is preserved. Second, we evaluate the effective coupling constants in ϕ^4 and Yukawa theory. We find that the IR effects from gravitons preserve the Lorentz invariance. The effective coupling constants are found to decrease with cosmic expansion with definite scaling exponents. In Section 4, we vary the gauge parameter of the graviton propagator to investigate the gauge dependences of the results obtained in Section 3. We show that the relative scaling exponents of the couplings are gauge invariant and observable. We conclude with discussions in Section 5.

2 Gravitational field in de Sitter space

In this section, we compute the gravitation propagator in de Sitter (dS) space. In the Poincaré coordinate, the metric in dS space is

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2, \quad a(t) = e^{Ht}, \tag{2.1}$$

where the dimension of dS space is taken as $D = 4$ and H is the Hubble constant. In the conformally flat coordinate,

$$(g_{\mu\nu})_{\text{dS}} = a^2(\tau)\eta_{\mu\nu}, \quad a(\tau) = -\frac{1}{H\tau}. \tag{2.2}$$

Here the conformal time τ ($-\infty < \tau < 0$) is related to the cosmic time t as $\tau \equiv -\frac{1}{H}e^{-Ht}$. We assume that de Sitter space begins at an initial time t_i with a finite spacial extension. After a sufficient exponential expansion, the de Sitter space is well described locally by the above metric irrespective of the spacial topology. The metric is invariant under the scaling transformation.

$$\tau \to c\tau, \quad x^i \to cx^i. \tag{2.3}$$

It is a part of the $SO(1,4)$ de Sitter symmetry.

In dealing with the quantum fluctuation whose background is dS space, we adopt the following parametrization:

$$g_{\mu\nu} = \Omega^2(x)\tilde{g}_{\mu\nu}, \ \Omega(x) = a(\tau)e^{\kappa w(x)}, \tag{2.4}$$

$$\det \tilde{g}_{\mu\nu} = -1, \ \tilde{g}_{\mu\nu} = (e^{\kappa h(x)})_{\mu\nu}, \tag{2.5}$$

where κ is defined by the Newton's constant G as $\kappa^2 = 16\pi G$. To satisfy (2.5), $h_{\mu\nu}$ is traceless

$$\eta^{\mu\nu}h_{\mu\nu} = 0. \tag{2.6}$$

By using this parametrization, the components of the Einstein action are written as follows. We keep a parameter D to specify the dimension for generality:

$$\sqrt{-g} = \Omega^D, \tag{2.7}$$

$$R = \Omega^{-2}\tilde{R} - 2(D-1)\Omega^{-3}\tilde{g}^{\mu\nu}\nabla_\mu\partial_\nu\Omega - (D-1)(D-4)\Omega^{-4}\tilde{g}^{\mu\nu}\partial_\mu\Omega\partial_\nu\Omega, \tag{2.8}$$

where $\tilde{R}$ is the Ricci scalar constructed from $\tilde{g}_{\mu\nu}$

$$\tilde{R} = -\partial_\mu\partial_\nu\tilde{g}^{\mu\nu} - \frac{1}{4}\tilde{g}^{\mu\nu}\tilde{g}^{\rho\sigma}\tilde{g}^{\alpha\beta}\partial_\mu\tilde{g}_{\rho\alpha}\partial_\nu\tilde{g}_{\sigma\beta} + \frac{1}{2}\tilde{g}^{\mu\nu}\tilde{g}^{\rho\sigma}\tilde{g}^{\alpha\beta}\partial_\mu\tilde{g}_{\sigma\alpha}\partial_\rho\tilde{g}_{\nu\beta}. \tag{2.9}$$

From (2.7) and (2.8), the lagrangian of gravity is

$$\begin{aligned}\mathcal{L}_{\text{gravity}} &= \frac{1}{\kappa^2}\sqrt{-g}\big[R - (D-1)(D-2)H^2\big] \\ &= \frac{1}{\kappa^2}\big[\Omega^{D-2}\tilde{R} + (D-1)(D-2)\Omega^{D-4}\tilde{g}^{\mu\nu}\partial_\mu\Omega\partial_\nu\Omega - (D-1)(D-2)H^2\Omega^D\big].\end{aligned} \tag{2.10}$$

Note that the lagrangian density is defined including $\sqrt{-g}$ in this paper.

In order to fix the gauge degrees from general coordinate invariance

$$\begin{aligned} x'_\mu &= x_\mu + \varepsilon_\mu, \\ g'_{\mu\nu} &= g_{\mu\nu} - g_{\mu\rho}\partial_\nu\varepsilon^\rho - g_{\nu\rho}\partial_\mu\varepsilon^\rho - \partial_\rho g_{\mu\nu}\varepsilon^\rho, \end{aligned} \tag{2.11}$$

we introduce the gauge fixing term [4]

$$\begin{aligned}\mathcal{L}_{\text{GF}} &= -\frac{1}{2}a^{D-2}\eta^{\mu\nu}F_\mu F_\nu, \\ F_\mu &= \partial_\rho h_\mu^{\ \rho} - (D-2)\partial_\mu w + (D-2)h_\mu^{\ \rho}\partial_\rho\log a + 2(D-2)w\partial_\mu\log a.\end{aligned} \tag{2.12}$$

The corresponding ghost term at the quadratic level is

$$\begin{aligned}\mathcal{L}_{\text{ghost}} =& - a^{D-2}\partial_\sigma\bar{b}^\mu\eta^{\sigma\nu}\big\{\eta_{\mu\rho}\partial_\nu + \eta_{\nu\rho}\partial_\mu + 2\eta_{\mu\nu}\partial_\rho(\log a)\big\}b^\rho \\ &+ \partial_\mu(a^{D-2}\bar{b}^\mu)\eta^{\rho\sigma}\big\{\eta_{\rho\nu}\partial_\sigma + \eta_{\rho\sigma}\partial_\nu(\log a)\big\}b^\nu,\end{aligned} \tag{2.13}$$

where b^μ is a ghost field and $\bar{b}^\mu$ is an anti-ghost field. From (2.9), (2.10), (2.12) and (2.13), the quadratic part of the total Lagrangian density is

$$\begin{aligned}\mathcal{L}_{\text{quadratic}} = a^{D-2}\Big\{&\frac{1}{2}D(D-2)\eta^{\mu\nu}\partial_\mu w\partial_\nu w - \frac{D}{4(D-1)}\eta^{\mu\nu}\partial_\mu h^{00}\partial_\nu h^{00} - \frac{1}{4}\eta^{\mu\nu}\partial_\mu\tilde{h}^i_{\ j}\partial_\nu\tilde{h}^j_{\ i} \\ &+ \frac{1}{2}\eta^{\mu\nu}\partial_\mu h^{0i}\partial_\nu h^{0i} + \eta^{\mu\nu}\partial_\mu\bar{b}^0\partial_\nu b^0 - \eta^{\mu\nu}\partial_\mu\bar{b}^i\partial_\nu b^i \\ &+ a^D H^2\big\{-2(D-2)w^2 + 2(D-2)wh^{00} - \frac{1}{2}(D-2)h^{00}h^{00} \\ &+ \frac{1}{2}(D-2)h^{0i}h^{0i} + (D-2)\bar{b}^0 b^0\big\}.\end{aligned} \tag{2.14}$$

Here we have decomposed $h^i{}_j,\ i,j=1,\cdots,D-1$ into a trace part and a traceless part

$$h^i{}_j=\tilde{h}^i{}_j+\frac{1}{D-1}h^k{}_k\delta^i{}_j=\tilde{h}^i{}_j+\frac{1}{D-1}h^{00}\delta^i{}_j. \tag{2.15}$$

(2.14) is diagonalized as

$$\begin{aligned}\mathcal{L}_{\text{quadratic}}=a^D\Big[&\frac{1}{2}a^{-2}\eta^{\mu\nu}\partial_\mu X\partial_\nu X-\frac{1}{4}a^{-2}\eta^{\mu\nu}\partial_\mu\tilde{h}^i{}_j\partial_\nu\tilde{h}^j{}_i-a^{-2}\eta^{\mu\nu}\partial_\mu\bar{b}^i\partial_\nu b^i\\&+\frac{1}{2}a^{-2}\eta^{\mu\nu}\partial_\mu h^{0i}\partial_\nu h^{0i}+\frac{1}{2}(D-2)H^2h^{0i}h^{0i}\\&-\frac{1}{2}a^{-2}\eta^{\mu\nu}\partial_\mu Y\partial_\nu Y-(D-3)H^2Y^2\\&+a^{-2}\eta^{\mu\nu}\partial_\mu\bar{b}^0\partial_\nu b^0+(D-2)H^2\bar{b}^0b^0\Big],\end{aligned} \tag{2.16}$$

where $X,\ Y$ are

$$X=(D-2)\sqrt{\frac{D-1}{D-3}}w-\frac{1}{\sqrt{(D-1)(D-3)}}h^{00},\quad Y=\sqrt{\frac{D-2}{2(D-3)}}(h^{00}-2w). \tag{2.17}$$

(2.16) contains three types of fields, massless and minimally coupled fields: $X,h^i{}_j,b^i,\bar{b}^i$, massless conformally coupled fields: $h^{0i},b^0,\bar{b}^0$, and Y.

We restrict to the $D=4$ case in the subsequent discussion. In this case, Y corresponds with a massless conformally coupled field. We list gravitation propagators as below

$$\begin{aligned}\langle X(x)X(x')\rangle&=-\langle\varphi(x)\varphi(x')\rangle,\\\langle\tilde{h}^i{}_j(x)\tilde{h}^k{}_l(x')\rangle&=(\delta^{ik}\delta_{jl}+\delta^i{}_l\delta_j{}^k-\frac{2}{3}\delta^i{}_j\delta^k{}_l)\langle\varphi(x)\varphi(x')\rangle,\\\langle b^i(x)\bar{b}^j(x')\rangle&=\delta^{ij}\langle\varphi(x)\varphi(x')\rangle,\end{aligned} \tag{2.18}$$

$$\begin{aligned}\langle h^{0i}(x)h^{0j}(x')\rangle&=-\delta^{ij}\langle\phi(x)\phi(x')\rangle,\\\langle Y(x)Y(x')\rangle&=\langle\phi(x)\phi(x')\rangle,\\\langle b^0(x)\bar{b}^0(x')\rangle&=-\langle\phi(x)\phi(x')\rangle,\end{aligned} \tag{2.19}$$

Here φ denotes a massless and minimally coupled scalar field and ϕ denotes a massless conformally coupled scalar field

$$\langle\varphi(x)\varphi(x')\rangle=\frac{H^2}{4\pi^2}\Big\{\frac{1}{y}-\frac{1}{2}\log y+\frac{1}{2}\log a(\tau)a(\tau')+1-\gamma\Big\}, \tag{2.20}$$

$$\langle\phi(x)\phi(x')\rangle=\frac{H^2}{4\pi^2}\frac{1}{y}, \tag{2.21}$$

where γ is Euler's constant and y is the dS invariant distance

$$y=\frac{-(\tau-\tau')^2+(\mathbf{x}-\mathbf{x}')^2}{\tau\tau'}. \tag{2.22}$$

It should be noted that the propagator for a massless and minimally coupled scalar field has the dS symmetry breaking logarithmic term: $\log a(\tau)a(\tau')$. To explain what causes the dS symmetry breaking, we recall the wave function for a massless and minimally coupled field

$$\phi_{\mathbf{p}}(x) = \frac{H\tau}{\sqrt{2p}}(1 - i\frac{1}{p\tau})e^{-ip\tau+i\mathbf{p}\cdot\mathbf{x}}. \tag{2.23}$$

Inside the cosmological horizon $P \equiv p/a(\tau) \gg H \Leftrightarrow p|\tau| \gg 1$, this wave function approaches to that in Minkowski space up to a cosmic scale factor

$$\phi_{\mathbf{p}}(x) \sim \frac{H\tau}{\sqrt{2p}}e^{-ip\tau+\mathbf{p}\cdot\mathbf{x}}. \tag{2.24}$$

On the other hand, the behavior outside the cosmological horizon $P \ll H$ is

$$\phi_{\mathbf{p}}(x) \sim \frac{H}{\sqrt{2p^3}}e^{i\mathbf{p}\cdot\mathbf{x}}. \tag{2.25}$$

The IR behavior indicates that the corresponding propagator has a scale invariant spectrum. As a direct consequence of it, the propagator has a logarithmic divergence from the IR contributions in the infinite volume limit.

To regularize the IR divergence, we introduce an IR cut-off ε_0 which fixes the minimum value of the comoving momentum

$$\int_{\varepsilon_0 a^{-1}(\tau)}^{H} dP. \tag{2.26}$$

With this prescription, more degrees of freedom (d.o.f.) go out the cosmological horizon with cosmic evolution. Due to the increase, the propagator acquires the growing time dependence which spoils the dS symmetry. In tribute to its origin, we call the dS symmetry breaking term the IR logarithm. Physically speaking, $1/\varepsilon_0$ is recognized as an initial size of universe when the exponential expanding starts. For simplicity, we set $\varepsilon_0 = H$ in (2.20).

As there is explicit time dependence in the propagator, physical quantities can acquire time dependence through the quantum loop corrections. We call them the quantum IR effects in dS space. By the power counting of the IR logarithms in quantum gravity, the leading IR effects at n-loop level is estimated as $(\kappa^2 H^2 \log a(\tau))^n$. It indicates that even if $\kappa^2 H^2 \ll 1$, the quantum effects can eventually grow up to the tree level magnitude . This is the reason why we focus on the quantum IR effects in dS space. Before concluding this section, we introduce an approximation. Focusing on the IR effects, we can neglect conformally coupled modes of gravity since they do not induce the IR logarithm. By applying this approximation, we can identify the following two modes as

$$h^{00} \simeq 2w \simeq \frac{\sqrt{3}}{2}X. \tag{2.27}$$

3 Quantum equation of motion

In the preceding section, we have reviewed that gravitational field contains massless and minimally coupled modes and the corresponding propagator is time dependent due to the

increase of d.o.f. outside the cosmological horizon. In this section, we investigate how the quantum IR effects from gravitons influence the local dynamics of the matter fields. More specifically, we evaluate the quantum equation of motion for matter fields including the quantum IR effects from gravitons.

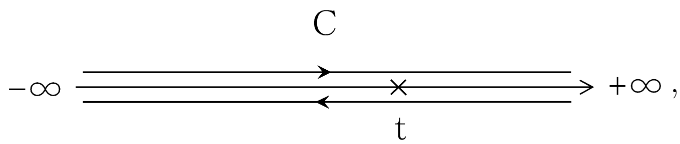
To begin with, we review how to derive the quantum equation of motion on a time dependent background [14]. Let us represent the vacuum at $t \to -\infty$ as $|in\rangle$, and $t \to \infty$ as $|out\rangle$. In the Feynman-Dyson formalism on a flat background, it is presumed that $|out\rangle$ is equal to $|in\rangle$ up to a phase factor. On the other hand, we can't prefix $|out\rangle$ in de Sitter space. The correct strategy is to evaluate vacuum expectation values (vev) with respect to $|in\rangle$:

$$\langle \mathcal{O}_H(x) \rangle = \langle in|T_C[U(-\infty,\infty)U(\infty,-\infty)\mathcal{O}_I(x)]|in\rangle. \tag{3.1}$$

where $\mathcal{O}_H$ and $\mathcal{O}_I$ denote the operators in the Heisenberg and the interaction pictures respectively. $U(t_1, t_2)$ is the time translation operator in the interaction picture

$$U(t_1,t_2) = \exp\big\{i\int_{t_2}^{t_1} d^4x\ \delta\mathcal{L}_I(x)\big\}. \tag{3.2}$$

$\delta\mathcal{L}$ denotes the interaction term of the lagrangian. It is crucial that the operator ordering T_C specified by the following path is adopted here



(3.3)

$$\int_C dt = \int_{-\infty}^{\infty} dt_+ - \int_{-\infty}^{\infty} dt_-.$$

We call it the Schwinger-Keldysh formalism. Since there are two time indices $+,-$ in this formalism, the propagator has four components

$$\begin{pmatrix} \langle\varphi_+(x)\varphi_+(x')\rangle & \langle\varphi_+(x)\varphi_-(x')\rangle \\ \langle\varphi_-(x)\varphi_+(x')\rangle & \langle\varphi_-(x)\varphi_-(x')\rangle \end{pmatrix} = \begin{pmatrix} \langle T\varphi(x)\varphi(x')\rangle & \langle\varphi(x')\varphi(x)\rangle \\ \langle\varphi(x)\varphi(x')\rangle & \langle\tilde{T}\varphi(x)\varphi(x')\rangle \end{pmatrix}, \tag{3.4}$$

where $\tilde{T}$ denotes the anti-time ordering.

We introduce the external source J_+, J_- for each path and evaluate

$$Z[J_+,J_-] = \langle in|T_C[U(-\infty,\infty)U(\infty,-\infty)\exp\big\{i\int d^4x\ (J_+\varphi_+ - J_-\varphi_-)\big\}]|in\rangle. \tag{3.5}$$

The generating functional for the connected Green's functions is

$$iW[J_+,J_-] = \log Z[J_+,J_-]. \tag{3.6}$$

We define the classical field as

$$\hat{\varphi}_A(x) = c_{AB}\frac{\delta W[J_+,J_-]}{\delta J_B(x)}, \quad A,B = +,-, \tag{3.7}$$

$$c_{AB} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{3.8}$$

By taking the limit $J_+ = J_- = J$ in (3.7), we obtain the vev of φ where the action contains the additional $J\varphi$ term

$$\langle\varphi(x)\rangle|_{J\varphi} = \hat{\varphi}_+(x)|_{J_+=J_-=J} = \hat{\varphi}_-(x)|_{J_+=J_-=J}. \tag{3.9}$$

Finally, we turn off the source term $J = 0$

$$\langle\varphi(x)\rangle = \frac{\delta W[J_+, J_-]}{\delta J_+(x)}\Big|_{J_+=J_-=0} = -\frac{\delta W[J_+, J_-]}{\delta J_-(x)}\Big|_{J_+=J_-=0}. \tag{3.10}$$

The effective action is obtained after the Legendre transformation

$$\Gamma[\hat{\varphi}_+, \hat{\varphi}_-] = W[J_+, J_-] - \int d^4x \ (J_+\hat{\varphi}_+ - J_-\hat{\varphi}_-), \tag{3.11}$$

where $J_{+,-}$ are given by $\hat{\varphi}_{+,-}$ as follows

$$J_A(x) = -c_{AB}\frac{\delta\Gamma[\hat{\varphi}_+, \hat{\varphi}_-]}{\delta\hat{\varphi}_B(x)}. \tag{3.12}$$

From (3.9) (3.12), we obtain in the limit $\hat{\varphi}_+ = \hat{\varphi}_- = \hat{\varphi}$

$$J(x) = -\frac{\delta\Gamma[\hat{\varphi}_+, \hat{\varphi}_-]}{\delta\hat{\varphi}_+(x)}\Big|_{\hat{\varphi}_+=\hat{\varphi}_-=\hat{\varphi}} = \frac{\delta\Gamma[\hat{\varphi}_+, \hat{\varphi}_-]}{\delta\hat{\varphi}_-(x)}\Big|_{\hat{\varphi}_+=\hat{\varphi}_-=\hat{\varphi}}. \tag{3.13}$$

In the absence of the external source, the exact equation of motion is obtained including quantum effects

$$\frac{\delta\Gamma[\hat{\varphi}_+, \hat{\varphi}_-]}{\delta\hat{\varphi}_+(x)}\Big|_{\hat{\varphi}_+=\hat{\varphi}_-=\hat{\varphi}} = -\frac{\delta\Gamma[\hat{\varphi}_+, \hat{\varphi}_-]}{\delta\hat{\varphi}_-(x)}\Big|_{\hat{\varphi}_+=\hat{\varphi}_-=\hat{\varphi}} = 0. \tag{3.14}$$

We call these identities quantum equation of motion in this paper. In the following subsections, we evaluate these identities in concrete models to understand quantum IR effects from gravitons to matter field dynamics at the one-loop level.

3.1 Free field theories

Let us investigate the effects of soft gravitons on the local dynamics of a free scalar and Dirac field. First we investigate a massless conformally coupled scalar field. The corresponding action is

$$S = \int \sqrt{-g}d^4x\big[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{12}R\phi^2\big]. \tag{3.15}$$

For convenience, we redefine the matter field $\Omega\phi \to \phi$:

$$S = \int d^4x \big[-\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{12}\tilde{R}\phi^2 \big]. \tag{3.16}$$

To obtain the quantum equation of the matter field, we decompose it into the classical field and the quantum fluctuation

$$\phi \to \hat{\phi} + \phi. \tag{3.17}$$

By differentiating (3.16) with respect to $\hat{\phi}_+$ as in (3.14), we obtain the quantum equation of motion. The quantum equation of motion up to the one-loop level is expressed as

$$\begin{aligned} &\partial^2\hat{\phi}(x) + \frac{1}{2}\kappa^2\partial_\mu(\langle (h^{\mu\rho})_+(x)(h_\rho{}^\nu)_+(x)\rangle\partial_\nu\hat{\phi}(x)) \\ &- i\kappa^2\partial_\mu \int d^4x'\ c_{AB}\langle (h^{\mu\nu})_+(x)(h^{\rho\sigma})_A(x')\rangle\langle \partial_\nu\phi_+(x)\partial'_\rho\phi_B(x')\rangle\partial'_\sigma\hat{\phi}(x') \simeq 0, \end{aligned} \tag{3.18}$$

where $\partial^2 = \eta^{\mu\nu}\partial_\mu\partial_\nu$ and we have taken the limit $\hat{\phi}_+ = \hat{\phi}_- = \hat{\phi}$. Furthermore, we have neglected the contribution from differentiated gravitational field. It is because we focus on the IR logarithms and the differentiated gravitational field doesn't induce them. Since we consider the dynamics of matter fields whose momentum scale is larger than the Hubble scale, there is no loss of generality to assume that the scalar field is conformally coupled to gravity.

From (2.18) and (2.27), we have only to focus on the following propagators to extract massless and minimally coupled modes from gravity

$$\begin{aligned} \langle h^{00}(x)h^{00}(x')\rangle &\simeq -\frac{3}{4}\langle\varphi(x)\varphi(x')\rangle, \\ \langle h^{00}(x)h^i{}_j(x')\rangle &\simeq -\frac{1}{4}\delta^i{}_j\langle\varphi(x)\varphi(x')\rangle, \\ \langle h^i{}_j(x)h^k{}_l(x')\rangle &\simeq (\delta^{ik}\delta_{jl} + \delta^i{}_l\delta_j{}^k - \frac{3}{4}\delta^i{}_j\delta^k{}_l)\langle\varphi(x)\varphi(x')\rangle. \end{aligned} \tag{3.19}$$

By adopting an ultra-violet (UV) regularization, the propagator at coincident point is estimated as follows

$$\langle\varphi(x)\varphi(x)\rangle = (\text{UV divergent const}) + \frac{H^2}{4\pi^2}\log a(\tau). \tag{3.20}$$

Henceforth we focus on the time dependent dS symmetry breaking part

$$\langle\varphi(x)\varphi(x)\rangle \simeq \frac{H^2}{4\pi^2}\log a(\tau). \tag{3.21}$$

On the other hand, the matter field contains only a conformally coupled mode and so we can use the exact propagator

$$\langle\phi(x)\phi(x')\rangle = \frac{1}{4\pi^2}\frac{1}{\Delta x^2}, \tag{3.22}$$

where $\Delta x^\mu \equiv x^\mu - x'^\mu,\ \Delta x^2 \equiv \eta_{\mu\nu}\Delta x^\mu \Delta x^\nu$. It should be noted that we have redefined the matter field. In (2.20) and (3.22), the Schwinger-Keldysh indices are assigned as follows

$$y_{AB} = H^2 a(\tau)a(\tau')\Delta x^2_{AB}, \quad A, B = \pm, \tag{3.23}$$

$$\begin{aligned}
\Delta x^2_{++} &= -(|\tau - \tau'| - ie)^2 + (\mathbf{x} - \mathbf{x}')^2, \\
\Delta x^2_{+-} &= -(\tau - \tau' + ie)^2 + (\mathbf{x} - \mathbf{x}')^2, \\
\Delta x^2_{-+} &= -(\tau - \tau' - ie)^2 + (\mathbf{x} - \mathbf{x}')^2, \\
\Delta x^2_{--} &= -(|\tau - \tau'| + ie)^2 + (\mathbf{x} - \mathbf{x}')^2,
\end{aligned} \tag{3.24}$$

where e is a positive infinitesimal quantity.

From (3.19) and (3.21), the second term in (3.18) is evaluated as

$$\frac{1}{2}\kappa^2 \partial_\mu(\langle (h^{\mu\rho})_+(x)(h_\rho{}^\nu)_+(x)\rangle \partial_\nu \hat{\phi}(x)) \simeq \frac{\kappa^2 H^2}{4\pi^2}\log a(\tau)\{\frac{3}{8}\partial_0^2 + \frac{13}{8}\partial_i^2\}\hat{\phi}(x). \tag{3.25}$$

To evaluate the third term in (3.18), we need to perform the following integration

$$\begin{aligned}
&- i\kappa^2 \partial_\mu \int d^4x'\ c_{AB}\langle \varphi_+(x)\varphi_A(x')\rangle \langle \partial_\nu \phi_+(x)\partial'_\rho \phi_B(x')\rangle \partial'_\sigma \hat{\phi}(x') \\
&\simeq i\kappa^2 \partial_\mu \partial_\nu \partial_\rho \partial_\sigma \int d^4x'\ c_{AB}\langle \varphi_+(x)\varphi_A(x')\rangle \langle \phi_+(x)\phi_B(x')\rangle \hat{\phi}(x').
\end{aligned} \tag{3.26}$$

Here we have not differentiated $\langle \varphi(x)\varphi(x')\rangle$ since it doesn't induce the IR logarithm. In order to evaluate the coefficient of $\log a(\tau)$, we may adopt the following approximation :

$$\begin{aligned}
&\int d^4x'\ c_{AB}\langle \varphi_+(x)\varphi_A(x')\rangle \langle \phi_+(x)\phi_B(x')\rangle \hat{\phi}(x') \\
&\simeq \langle \varphi_+(x)\varphi_+(x)\rangle \int d^4x' \big[\langle \phi_+(x)\phi_+(x')\rangle - \langle \phi_+(x)\phi_-(x')\rangle\big]\hat{\phi}(x').
\end{aligned} \tag{3.27}$$

This approximation method has been introduced in Yukawa theory and scalar QED [15, 16, 17].

We argue that the IR logarithms appear only in the local terms. As we can observe in (2.20), the logarithmic term in the propagator only depends on the relative coordinates. So they do not lead to an explicit time dependence unless in a singular limit, namely local terms. Therefore we extract local terms in quantum equation of motion. To do that, we need to expand $\hat{\phi}$ up to the second order

$$\hat{\phi}(x') \to \hat{\phi}(x) - \partial_\alpha \hat{\phi}(x)\Delta x^\alpha + \frac{1}{2}\partial_\alpha \partial_\beta \hat{\phi}(x)\Delta x^\alpha \Delta x^\beta. \tag{3.28}$$

From (3.21), (3.22), (3.27) and (3.28), (3.26) is written as

$$\begin{aligned}
i\frac{\kappa^2 H^2}{16\pi^4}\log a(\tau)\partial_\mu \partial_\nu \partial_\rho \partial_\sigma \int d^4x' \Big\{ \big[\frac{1}{\Delta x^2_{++}} - \frac{1}{\Delta x^2_{+-}}\big]\hat{\phi}(x) - \big[\frac{\Delta x^\alpha}{\Delta x^2_{++}} - \frac{\Delta x^\alpha}{\Delta x^2_{+-}}\big]\partial_\alpha \hat{\phi}(x) \\
+ \frac{1}{2}\big[\frac{\Delta x^\alpha \Delta x^\beta}{\Delta x^2_{++}} - \frac{\Delta x^\alpha \Delta x^\beta}{\Delta x^2_{+-}}\big]\partial_\alpha \partial_\beta \hat{\phi}(x) \Big\}.
\end{aligned} \tag{3.29}$$

To investigate the kinetic term, we need to perform the following integrations. We list the results below

$$\partial_\alpha\partial_\beta\int d^4x'\big[\frac{1}{\Delta x^2_{++}}-\frac{1}{\Delta x^2_{+-}}\big]=-4i\pi^2\delta_\alpha^{\ 0}\delta_\beta^{\ 0}, \tag{3.30}$$

$$\partial_\beta\partial_\gamma\partial_\delta\int d^4x'\big[\frac{\Delta x_\alpha}{\Delta x^2_{++}}-\frac{\Delta x_\alpha}{\Delta x^2_{+-}}\big]=8i\pi^2\delta_\alpha^{\ 0}\delta_\beta^{\ 0}\delta_\gamma^{\ 0}\delta_\delta^{\ 0}, \tag{3.31}$$

$$\begin{aligned}&\partial_\gamma\partial_\delta\partial_\varepsilon\partial_\eta\int d^4x'\big[\frac{\Delta x_\alpha\Delta x_\beta}{\Delta x^2_{++}}-\frac{\Delta x_\alpha\Delta x_\beta}{\Delta x^2_{+-}}\big]\\ =&-32i\pi^2\delta_\alpha^{\ 0}\delta_\beta^{\ 0}\delta_\gamma^{\ 0}\delta_\delta^{\ 0}\delta_\varepsilon^{\ 0}\delta_\eta^{\ 0}-8i\pi^2\eta_{\alpha\beta}\delta_\gamma^{\ 0}\delta_\delta^{\ 0}\delta_\varepsilon^{\ 0}\delta_\eta^{\ 0}.\end{aligned} \tag{3.32}$$

We explain how to derive them in Appendix A. In total, (3.26) induces the following kinetic term with the IR logarithm

$$\begin{aligned}\frac{\kappa^2H^2}{4\pi^2}\log a(\tau)\Big\{&\delta_\mu^{\ 0}\delta_\nu^{\ 0}\partial_\rho\partial_\sigma+\delta_\mu^{\ 0}\delta_\rho^{\ 0}\partial_\nu\partial_\sigma+\delta_\mu^{\ 0}\delta_\sigma^{\ 0}\partial_\nu\partial_\rho+\delta_\nu^{\ 0}\delta_\rho^{\ 0}\partial_\mu\partial_\sigma+\delta_\nu^{\ 0}\delta_\sigma^{\ 0}\partial_\mu\partial_\rho+\delta_\rho^{\ 0}\delta_\sigma^{\ 0}\partial_\mu\partial_\nu\\ &-2(\delta_\mu^{\ 0}\delta_\nu^{\ 0}\delta_\rho^{\ 0}\partial_\sigma+\delta_\mu^{\ 0}\delta_\nu^{\ 0}\delta_\sigma^{\ 0}\partial_\rho+\delta_\mu^{\ 0}\delta_\rho^{\ 0}\delta_\sigma^{\ 0}\partial_\nu+\delta_\nu^{\ 0}\delta_\rho^{\ 0}\delta_\sigma^{\ 0}\partial_\mu)\partial_0\\ &+4\delta_\mu^{\ 0}\delta_\nu^{\ 0}\delta_\rho^{\ 0}\delta_\sigma^{\ 0}\partial_0^2+\delta_\mu^{\ 0}\delta_\nu^{\ 0}\delta_\rho^{\ 0}\delta_\sigma^{\ 0}\partial^2\Big\}\hat{\phi}(x).\end{aligned} \tag{3.33}$$

From (3.19) and (3.33), the third term in (3.18) leads to

$$\frac{\kappa^2H^2}{4\pi^2}\log a(\tau)\big\{-\frac{3}{4}\partial_0^2-\frac{5}{4}\partial_i^2\big\}\hat{\phi}(x). \tag{3.34}$$

From (3.25) and (3.34), the quantum equation of motion of a scalar field including the one-loop correction from soft gravitons is

$$\left(1+\frac{3\kappa^2H^2}{32\pi^2}\log a(\tau)\right)\partial^2\hat{\phi}(x). \tag{3.35}$$

Although each contribution (3.25), (3.34) breaks the Lorentz invariance, the total of them preserves it. The IR effect emerges just as an overall factor. Up to $\kappa^2H^2\log a(\tau)$, we can eliminate it by the following time dependent renormalization of a scalar field:

$$\phi\to Z_\phi\phi,\quad Z_\phi\simeq 1-\frac{3\kappa^2H^2}{64\pi^2}\log a(\tau). \tag{3.36}$$

We have checked that the same result is obtained in an exact calculation with the dimensional regularization. The IR logarithm originates from the dS symmetry breaking term in the graviton propagator in such a calculation. We also remark that the IR logarithm can be absorbed into the wave function renormalization factor even if we include a mass term as a perturbation.

Next we perform a parallel investigation with a Dirac field. The corresponding action is

$$S=\int\sqrt{-g}d^4x\ i\bar{\psi}e^\mu{}_a\gamma^a\nabla_\mu\psi, \tag{3.37}$$

where $e^\mu{}_a$ is a vierbein and γ^a is the gamma matrix:

$$\gamma^a\gamma^b+\gamma^b\gamma^a=-2\eta^{ab}. \tag{3.38}$$

The vierbein can be parametrized as

$$e^\mu{}_a=\Omega^{-1}\tilde{e}^\mu{}_a,\quad \tilde{e}^\mu{}_a=(e^{-\frac{\kappa}{2}h})^\mu{}_a. \tag{3.39}$$

In a similar way to (3.16), we redefine the matter field $\Omega^{\frac{3}{2}}\psi\to\psi$:

$$S=\int d^4x\ i\bar{\psi}\tilde{e}^\mu{}_a\gamma^a\tilde{\nabla}_\mu\psi. \tag{3.40}$$

By decomposing ψ into the classical field and the quantum fluctuation

$$\psi\to\hat{\psi}+\psi, \tag{3.41}$$

and differentiating (3.40) with respect to $\hat{\bar{\psi}}$, the quantum equation of motion up to the one-loop level is written as

$$\begin{aligned}&i\eta^\mu{}_a\gamma^a\partial_\mu\hat{\psi}(x)+i\frac{\kappa^2}{8}\langle(h^\mu{}_\rho)_+(x)(h^\rho{}_a)_+(x)\rangle\gamma^a\partial_\mu\hat{\psi}(x)\\&-i\frac{\kappa^2}{4}\int d^4x'\ c_{AB}\langle(h^\mu{}_a)_+(x)(h^\nu{}_b)_A(x')\rangle\gamma^a\langle\partial_\mu\psi_+(x)\bar{\psi}_B(x')\rangle\gamma^b\partial_\nu\hat{\psi}(x')\simeq 0.\end{aligned} \tag{3.42}$$

Here we have approximated $\tilde{\nabla}_\mu\simeq\partial_\mu$ since the differentiated gravitational field doesn't contribute to the IR logarithm.

By substituting the identity $\langle\psi(x)\bar{\psi}(x')\rangle=i\eta^\rho{}_c\gamma^c\partial_\rho\langle\phi(x)\phi(x')\rangle$, (3.42) is written as

$$\begin{aligned}&i\eta^\mu{}_a\gamma^a\partial_\mu\hat{\psi}(x)+i\frac{\kappa^2}{8}\langle(h^\mu{}_\rho)_+(x)(h^\rho{}_a)_+(x)\rangle\gamma^a\partial_\mu\hat{\psi}(x)\\&+\frac{\kappa^2}{4}\eta^\rho{}_c\int d^4x'\ c_{AB}\langle(h^\mu{}_a)_+(x)(h^\nu{}_b)_A(x')\rangle\gamma^a\gamma^c\langle\partial_\mu\partial_\rho\phi_+(x)\phi_B(x')\rangle\gamma^b\partial_\nu\hat{\psi}(x')\simeq 0.\end{aligned} \tag{3.43}$$

From (3.19) and (3.21), the second term in (3.43) is evaluated as

$$i\frac{\kappa^2}{8}\langle(h^\mu{}_\rho)_+(x)(h^\rho{}_a)_+(x)\rangle\gamma^a\partial_\mu\hat{\psi}(x)\simeq\frac{\kappa^2H^2}{4\pi^2}\log a(\tau)\times i\big\{-\frac{3}{32}\gamma^0\partial_0+\frac{13}{32}\gamma^i\partial_i\big\}\hat{\psi}(x). \tag{3.44}$$

To evaluate the third term in (3.43), we need to perform the following integration

$$\begin{aligned}&\frac{\kappa^2}{4}\eta^\rho{}_c\int d^4x'\ c_{AB}\langle\varphi_+(x)\varphi_A(x')\rangle\gamma^a\gamma^c\langle\partial_\mu\partial_\rho\phi_+(x)\phi_B(x')\rangle\gamma^b\partial_\nu\hat{\psi}(x')\\&\simeq\frac{\kappa^2}{4}\langle\varphi_+(x)\varphi_+(x)\rangle\eta^\rho{}_c\gamma^a\gamma^c\gamma^b\partial_\mu\partial_\nu\partial_\rho\int d^4x'\big[\langle\phi_+(x)\phi_+(x')\rangle-\langle\phi_+(x)\phi_-(x')\rangle\big]\hat{\psi}(x').\end{aligned} \tag{3.45}$$

Here we have neglected the differentiated gravitational fields and adopted the same approximation procedure with scalar field case.

Just like scalar field theory case, we need to evaluate local terms to estimate quantum IR effects due to soft gravitons. For such a purpose, we need to expand $\hat{\psi}(x')$ up to the first order

$$\hat{\psi}(x') \to \hat{\psi}(x) - \partial_\alpha \hat{\psi}(x)\Delta x^\alpha. \tag{3.46}$$

From (3.21), (3.22) and (3.46), (3.45) is written as

$$\begin{aligned}&\frac{\kappa^2 H^2}{64\pi^4}\log a(\tau)\eta^\rho{}_c\gamma^a\gamma^c\gamma^b\partial_\mu\partial_\nu\partial_\rho\int d^4x' \\ &\qquad\times\Big\{\big[\frac{1}{\Delta x^2_{++}}-\frac{1}{\Delta x^2_{+-}}\big]\hat{\psi}(x)-\big[\frac{\Delta x^\alpha}{\Delta x^2_{++}}-\frac{\Delta x^\alpha}{\Delta x^2_{+-}}\big]\partial_\alpha\hat{\psi}(x)\Big\}.\end{aligned} \tag{3.47}$$

By substituting (3.30) and (3.31) to (3.47), (3.45) is evaluated as

$$i\frac{\kappa^2 H^2}{4\pi^2}\log a(\tau)\eta^\rho{}_c\gamma^a\gamma^c\gamma^b\big\{-\frac{1}{4}(\delta_\mu{}^0\delta_\nu{}^0\partial_\rho+\delta_\mu{}^0\delta_\rho{}^0\partial_\nu+\delta_\nu{}^0\delta_\rho{}^0\partial_\mu)+\frac{1}{2}\delta_\mu{}^0\delta_\nu{}^0\delta_\rho{}^0\partial_0\big\}\hat{\psi}(x). \tag{3.48}$$

From (3.19) and (3.48), the third term in (3.43) is

$$\frac{\kappa^2 H^2}{4\pi^2}\log a(\tau)\times i\big\{\frac{3}{16}\gamma^0\partial_0-\frac{5}{16}\gamma^i\partial_i\big\}\hat{\psi}(x). \tag{3.49}$$

From (3.44) and (3.49), the quantum equation of motion of a Dirac field including the one-loop correction from soft gravitons is

$$\Big(1+\frac{3\kappa^2H^2}{128\pi^2}\log a(\tau)\Big)\times i\eta^\mu{}_a\gamma^a\partial_\mu\hat{\psi}(x). \tag{3.50}$$

Just like a scalar field, the IR effect from gravitons to a Dirac field preserves the Lorentz invariance. It can be eliminated by the following time dependent wave function renormalization of a Dirac field:

$$\psi\to Z_\psi\psi,\quad Z_\psi\simeq 1-\frac{3\kappa^2H^2}{256\pi^2}\log a(\tau). \tag{3.51}$$

We also remark again that the IR logarithm can be absorbed into the wave function renormalization factor even if we include a mass term as a perturbation.

We summarize our investigations in this subsection. Inside the cosmological horizon, the IR effects of the gravitons at the one-loop level preserve the Lorentz invariance both in a free scalar and Dirac field theory. We suspect that this is the case beyond the one-loop level in the both scalar and Dirac field cases. Although we think it is likely that soft graviton effects do not spoil Lorentz invariance of local physics, so far we have only demonstrated it by explicit calculations. We need to understand a mechanism to ensure it to all orders in perturbation theory. Our analysis has shown that the IR effects manifest as the overall factors of the kinetic terms. In free field theories, they can be renormalized away by a time dependent wave function renormalization. With interaction, the wave function renormalization contributes to the renormalization of the coupling constants. We investigate the IR effects in ϕ^4 theory and Yukawa theory in the next subsection.

3.2 ϕ^4 theory and Yukawa theory

Here we investigate the IR effects form gravitons to interacting field theories. As specific examples, we adopt ϕ^4 theory and Yukawa theory. Since the coupling constants are dimensionless, $\sqrt{-g}$ can be absorbed by the field redefinition $\Omega\phi \to \phi,\ \Omega^{\frac{3}{2}}\psi \to \psi$

$$\delta\mathcal{L}_4 = -\frac{\lambda}{4!}\phi^4, \tag{3.52}$$

$$\delta\mathcal{L}_Y = -g\phi\bar{\psi}\psi. \tag{3.53}$$

After the wave function renormalization (3.36), (3.51), the interaction terms are renormalized as

$$\delta\mathcal{L}_4 = -\frac{\lambda}{4!}Z_\phi^4\phi^4, \tag{3.54}$$

$$\delta\mathcal{L}_Y = -gZ_\phi Z_\psi^2\phi\bar{\psi}\psi. \tag{3.55}$$

First, we investigate ϕ^4 theory. Up to the one-loop level, the following nonlinear terms should be added to the left-hand side in (3.18)

$$\begin{aligned}
&-\frac{\lambda}{6}Z_\phi^4\hat{\phi}^3(x) + \frac{\lambda\kappa^2}{2}\int d^4x'd^4x''\ c_{AB}c_{CD}\langle(h^{\mu\nu})_+(x)(h^{\rho\sigma})_A(x')\rangle \\
&\qquad\qquad \times\langle\partial_\mu\partial_\nu\phi_+(x)\phi_C(x'')\rangle\langle\partial'_\rho\partial'_\sigma\phi_B(x')\phi_D(x'')\rangle\hat{\phi}(x')\hat{\phi}^2(x'') \\
&\qquad + \frac{\lambda\kappa^2}{2}\int d^4x'd^4x''\ c_{AB}c_{CD}\langle(h^{\mu\nu})_A(x')(h^{\rho\sigma})_C(x'')\rangle \\
&\qquad\qquad \times\langle\phi_+(x)\partial'_\mu\partial'_\nu\phi_B(x')\rangle\langle\phi_+(x)\partial''_\rho\partial''_\sigma\phi_D(x'')\rangle\hat{\phi}(x)\hat{\phi}(x')\hat{\phi}(x'').
\end{aligned} \tag{3.56}$$

Here we have performed the partial integration and neglected the differentiated gravitational field. The second and third terms denote the quantum correction to the vertex. The purpose in this subsection is to evaluate the effective coupling constant. To do that, we have only to extract the zeroth order of the classical fields in the Taylor expansion of the relative coordinates

$$\hat{\phi}(x')\hat{\phi}^2(x''),\ \hat{\phi}(x)\hat{\phi}(x')\hat{\phi}(x'') \to \hat{\phi}^3(x). \tag{3.57}$$

In a similar way to (3.27) and (3.45), to evaluate these integrals up to $\kappa^2H^2\log a(\tau)$, we may adopt the following approximation

$$\langle(h^{\mu\nu})_+(x)(h^{\rho\sigma})_+(x)\rangle\int d^4x'd^4x''. \tag{3.58}$$

In this approximation, only the following local term contributes to the remaining integrals

$$\langle\partial_\mu\partial_\nu\phi_+(x)\phi_+(x')\rangle = -\langle\partial_\mu\phi_+(x)\partial'_\nu\phi_+(x')\rangle \to -i\delta_\mu{}^0\delta_\nu{}^0\delta^{(4)}(x-x'). \tag{3.59}$$

As a result, (3.56) is evaluated as

$$\begin{aligned}
&-\frac{\lambda}{6}Z_\phi^4\hat{\phi}^3(x)-\lambda\kappa^2\langle(h^{00})_+(x)(h^{00})_+(x)\rangle\hat{\phi}^3(x) \\
\simeq&-\frac{1}{6}\lambda\big\{1-\frac{21\kappa^2H^2}{16\pi^2}\log a(\tau)\big\}\hat{\phi}^3(x).
\end{aligned} \tag{3.60}$$

In the second line, we have substituted (3.19), (3.21) and (3.36). (3.60) implies that the effective coupling constant in ϕ^4 theory decreases with cosmic expansion under the influence of soft gravitons

$$\lambda_{\text{eff}}\simeq\lambda\big\{1-\frac{21\kappa^2H^2}{16\pi^2}\log a(\tau)\big\}. \tag{3.61}$$

Next, we investigate Yukawa theory. Up to the one-loop level, the following nonlinear terms should be added to the left-hand side in (3.42)

$$\begin{aligned}
-gZ_\phi Z_\psi^2\hat{\phi}(x)\hat{\psi}(x)+\frac{g\kappa^2}{4}&\int d^4x'd^4x''\ c_{AB}c_{CD}\langle(h^\mu{}_a)_+(x)(h^\nu{}_b)_A(x')\rangle \\
&\times\langle\gamma^a\partial_\mu\psi_+(x)\bar{\psi}_C(x'')\rangle\langle\psi_D(x'')\partial'_\nu\bar{\psi}_B(x')\rangle\hat{\phi}(x'')\gamma^b\hat{\psi}(x') \\
+i\frac{g\kappa^2}{2}&\int d^4x'd^4x''\ c_{AB}c_{CD}\langle(h^\mu{}_a)_+(x)(h^{\rho\sigma})_A(x')\rangle \\
&\times\langle\gamma^a\partial_\mu\psi_+(x)\bar{\psi}_C(x'')\rangle\langle\partial'_\rho\partial'_\sigma\phi_B(x')\phi_D(x'')\rangle\hat{\phi}(x')\hat{\psi}(x'') \\
-i\frac{g\kappa^2}{2}&\int d^4x'd^4x''\ c_{AB}c_{CD}\langle(h^\mu{}_a)_A(x')(h^{\rho\sigma})_C(x'')\rangle \\
&\times\langle\psi_+(x)\partial'_\mu\bar{\psi}_B(x')\rangle\langle\phi_+(x)\partial''_\rho\partial''_\sigma\phi_D(x'')\rangle\hat{\phi}(x'')\gamma^a\hat{\psi}(x').
\end{aligned} \tag{3.62}$$

To evaluate the effective coupling constant, we have only to extract the zeroth order of the classical fields in the Taylor expansion of the relative coordinates

$$\hat{\phi}(x'')\hat{\psi}(x'),\ \hat{\phi}(x')\hat{\psi}(x'')\to\hat{\phi}(x)\hat{\psi}(x). \tag{3.63}$$

By substituting $\langle\psi(x)\bar{\psi}(x')\rangle=i\eta^\nu{}_c\gamma^c\partial_\nu\langle\phi(x)\phi(x')\rangle$ and performing the parallel procedure with (3.58)-(3.59), (3.62) is evaluated as

$$\begin{aligned}
&-gZ_\phi Z_\psi^2\hat{\phi}(x)\hat{\psi}(x)-\frac{5\kappa^2}{4}\langle(h^{00})_+(x)(h^{00})_+(x)\rangle\hat{\phi}(x)\hat{\psi}(x) \\
\simeq&-g\big\{1-\frac{39\kappa^2H^2}{128\pi^2}\log a(\tau)\big\}\hat{\phi}(x)\hat{\psi}(x).
\end{aligned} \tag{3.64}$$

We find that the effective coupling constant decreases with cosmic expansion also in Yukawa theory:

$$g_{\text{eff}}\simeq g\big\{1-\frac{39\kappa^2H^2}{128\pi^2}\log a(\tau)\big\}. \tag{3.65}$$

As seen in (3.61) and (3.65), the gravitational fluctuations outside the cosmological horizon influence the physics inside the cosmological horizon at the one-loop level. By the power

counting of the IR logarithms, the leading IR effect at the n-loop level is estimated as of order $(\kappa^2 H^2 \log a(\tau))^n$. Since the perturbation theory is broken after $\kappa^2 H^2 \log a(\tau) \sim 1$, we need a non-perturbative method to understand its long term consequences. Of course we need to make sure that this is a physical effect. In order to clarify such an issue, we investigate the gauge dependence of these perturbative IR effects in the next section.

4 Gauge dependence

In the previous sections, we have investigated the IR effects with the gauge fixing term (2.12). It is important to investigate the gauge dependence of the obtained results. In this section, we adopt the following gauge fixing term with a parameter β:

$$\begin{aligned}\mathcal{L}_{\rm GF} &= -\frac{1}{2}a^{D-2}\eta^{\mu\nu}F_\mu F_\nu, \\ F_\mu &= \beta\partial_\rho h_\mu^{\ \rho} - \beta(D-2)\partial_\mu w + \frac{1}{\beta}(D-2)h_\mu^{\ \rho}\partial_\rho \log a + \frac{2}{\beta}(D-2)w\partial_\mu \log a.\end{aligned} \tag{4.1}$$

This gauge fixing term coincides with (2.12) at $\beta = 1$. For simplicity, we consider the case $\beta^2 - 1 \ll 1$ since the deformation from (2.12) can be investigated perturbatively. The deformation of the action at $\mathcal{O}(\beta^2-1)$ is

$$\begin{aligned}\delta\mathcal{L}_{\beta^2-1} \simeq -\frac{1}{2}(\beta^2-1)a^2\big[\ &\eta^{\mu\nu}\partial_\mu h^{00}\partial_\nu h^{00} - 3\partial_0 h^{00}\partial_0 h^{00} - \frac{5}{9}\partial_i h^{00}\partial_i h^{00} \\ &- \frac{4}{3}\partial_i h^{00}\partial_k \tilde{h}^{ki} + \partial_k \tilde{h}^k_{\ i}\partial_l \tilde{h}^{li}\big].\end{aligned} \tag{4.2}$$

Here we have set $D = 4$ and neglected massless conformally coupled modes. In addition, we have ignored ghost fields since they do not couple to matter fields. To investigate the gauge dependence, we evaluate the correction to the gravitational propagator from the additional term (4.2).

As seen in (3.27), (3.45) and (3.58), we only need to investigate the graviton propagator at the coincident point to evaluate the coefficient of $\kappa^2 H^2 \log a(\tau)$. From (3.19) and (4.2), the IR logarithm can emerge in the following propagators

$$\begin{aligned}\langle (h^{00})_+(x)(h^{00})_+(x)\rangle|_{\beta^2-1} \simeq &-i(\beta^2-1)\int d^4x'\ a^2(\tau')c_{AB} \\ &\times\big\{\ \eta^{\mu\nu}\langle (h^{00})_+(x)\partial'_\mu(h^{00})_A(x')\rangle\langle (h^{00})_+(x)\partial'_\nu(h^{00})_B(x')\rangle \\ &\quad - 3\langle (h^{00})_+(x)\partial'_0(h^{00})_A(x')\rangle\langle (h^{00})_+(x)\partial'_0(h^{00})_B(x')\rangle \\ &\quad - \frac{5}{9}\langle (h^{00})_+(x)\partial'_i(h^{00})_A(x')\rangle\langle (h^{00})_+(x)\partial'_i(h^{00})_B(x')\rangle\big\},\end{aligned} \tag{4.3}$$

$$\begin{aligned}\langle (h^{00})_+(x)(\tilde{h}^{ij})_+(x)\rangle|_{\beta^2-1} \simeq &-i(\beta^2-1)\int d^4x'\ a^2(\tau')c_{AB} \\ &\times\big\{-\frac{2}{3}\langle (h^{00})_+(x)\partial'_k(h^{00})_A(x')\rangle\langle (\tilde{h}^{ij})_+(x)\partial'_l(\tilde{h}^{lk})_B(x')\rangle\big\},\end{aligned} \tag{4.4}$$

$$\langle(\tilde{h}^{ij})_+(x)(\tilde{h}^{kl})_+(x)\rangle|_{\beta^2-1} \simeq -i(\beta^2-1)\int d^4x'\ a^2(\tau')c_{AB} \times\big\{\langle(\tilde{h}^{ij})_+(x)\partial'_m(\tilde{h}^{mp})_A(x')\rangle\langle(\tilde{h}^{kl})_+(x)\partial'_n(\tilde{h}^{np})_B(x')\rangle\big\}. \tag{4.5}$$

To evaluate them, we need to perform the integration involving a massless and minimally coupled field

$$-i(\beta^2-1)\int d^4x'\ a^2(\tau')c_{AB}\langle\varphi_+(x)\partial'_\mu\varphi_A(x')\rangle\langle\varphi_+(x)\partial'_\nu\varphi_B(x')\rangle. \tag{4.6}$$

In the integral, the following term of (2.20) contributes to the IR logarithm

$$\langle\varphi(x)\varphi(x')\rangle \simeq \frac{H^2}{8\pi^2}\big\{-\log y+\log a(\tau)a(\tau')\big\} = -\frac{H^2}{8\pi^2}\log H^2\Delta x^2. \tag{4.7}$$

By substituting (4.7) and using the following identity

$$\int d^4x'\ \frac{1}{\tau'^2}\Big[\frac{\Delta x_\mu\Delta x_\nu}{\Delta x^4_{++}}-\frac{\Delta x_\mu\Delta x_\nu}{\Delta x^4_{+-}}\Big] \simeq -4i\pi^2\log a(\tau)\big\{\delta_\mu{}^0\delta_\nu{}^0+\frac{1}{2}\eta_{\mu\nu}\big\}, \tag{4.8}$$

the integral (4.6) is evaluated as

$$-(\beta^2-1)\frac{H^2}{4\pi^2}\log a(\tau)\big\{\delta_\mu{}^0\delta_\nu{}^0+\frac{1}{2}\eta_{\mu\nu}\big\}. \tag{4.9}$$

We explain how to derive the identity (4.8) in Appendix A.

From (4.9), (4.3)-(4.5) are

$$\begin{aligned}
&\langle(h^{00})_+(x)(h^{00})_+(x)\rangle|_{\beta^2-1} \simeq -(\beta^2-1)\times-\frac{3}{4}\frac{H^2}{4\pi^2}\log a(\tau), \\
&\langle(h^{00})_+(x)(\tilde{h}^{ij})_+(x)\rangle|_{\beta^2-1} \simeq 0, \\
&\langle(\tilde{h}^{ij})_+(x)(\tilde{h}^{kl})_+(x)\rangle|_{\beta^2-1} \simeq -(\beta^2-1)\times(\delta^{ik}\delta^{jl}+\delta^{il}\delta^{jk}-\frac{2}{3}\delta^{ij}\delta^{kl})\frac{H^2}{4\pi^2}\log a(\tau).
\end{aligned} \tag{4.10}$$

We should note that the deformation of propagator is proportional to the original one. As a result, in the deformed gauge (4.1), we have only to replace the gravitational propagator as follows

$$\langle(h^{\mu\nu})_+(x)(h^{\rho\sigma})_+(x)\rangle \to \big\{1-(\beta^2-1)\big\}\langle(h^{\mu\nu})_+(x)(h^{\rho\sigma})_+(x)\rangle. \tag{4.11}$$

From this fact, we can conclude that the Lorentz invariance is preserved for a continuous β

$$Z_\phi \simeq 1-(2-\beta^2)\frac{3\kappa^2H^2}{64\pi^2}\log a(\tau), \tag{4.12}$$

$$Z_\psi \simeq 1-(2-\beta^2)\frac{3\kappa^2H^2}{256\pi^2}\log a(\tau). \tag{4.13}$$

However the effective couplings are found to be gauge dependent.

$$\lambda_{\text{eff}} \simeq \lambda\big\{1-(2-\beta^2)\frac{21\kappa^2H^2}{16\pi^2}\log a(\tau)\big\},$$
$$g_{\text{eff}} \simeq g\big\{1-(2-\beta^2)\frac{39\kappa^2H^2}{128\pi^2}\log a(\tau)\big\}. \tag{4.14}$$

Here we summarize our findings as follows. In the original gauge the dimensionless coupling constants λ and g are screened by soft gravitons and decrease with time in de Sitter space. They acquire nontrivial scaling exponents in de Sitter space as

$$\lambda_{\text{eff}} = \lambda f(t)^{\frac{21}{4}}, \quad g_{\text{eff}} = g f(t)^{\frac{39}{32}} \tag{4.15}$$

where $f(t) = 1-\frac{\kappa^2H^2}{4\pi^2}Ht$. In order to confirm that this is a physical effect, we have examined the gauge parameter dependence of these results using one parameter family of the graviton propagator. As it turns out $f(t)$ depends on a gauge parameter β as $f(t) = 1-(2-\beta^2)\frac{\kappa^2H^2}{4\pi^2}Ht$. Neverthless we observe that the following relative scaling relation is independent of a gauge parameter.

$$g_{\text{eff}} \sim (\lambda_{\text{eff}})^{\frac{13}{56}} \tag{4.16}$$

We point out that the situation here is analogous to the scaling exponents of the operators in two dimensional quantum gravity. The scaling exponents of the individual operators are gauge dependent. However the relative scaling exponents are gauge invariant [18, 19]. It is because there is no unique way to specify the scale there. Analogously in our case there is no unique way to specify the time as it depends on an observer. Here a sensible strategy is to pick a particular coupling and use its time evolution as a physical time. In this setting the relative scaling exponents measure the speed of the time evolution of the couplings in terms of a physical time. Although the choice of time is not unique, the relative scaling exponents are gauge independent and well defined. We still need to check the validity of this picture against large deformations of a gauge parameter.

5 Conclusion

The inflation theory postulates that almost scale invariant density perturbation is generated by quantum fluctuations in a de Sitter type space. The detection of almost scale invariant gravitational fluctuations is a crucial test of the inflation theory since it universally generates them. Thus the propagator of gravitons contains de Sitter symmetry breaking IR logarithms in de Sitter space.

In this paper we have investigated its physical implications on microscopic physics. Namely we have investigated the effects of super-horizon modes of gravitons on the dynamics of the sub-horizon modes of matter fields. By evaluating the kinetic terms of scalar and Dirac fields up to the one-loop level, we have found that the IR effects from gravitons preserve the Lorentz

invariance. In particular the velocity of massless particles remains universal irrespective of the spins. Furthermore they can be absorbed into the wave function renormalization of scalar and Dirac fields. S. B. Giddings and M. S. Sloth investigate the scalar two point function in the quenched approximation [20]. S. P. Miao and R. P. Woodard show that the IR effect on the Dirac field is recognized as the wave function renormalization [13]. Their results qualitatively agree with ours.

In the interacting field theory with dimensionless couplings, we find that the couplings become time dependent and scale with time. In such a situation any coupling can be chosen as a physical time. Then the relative scaling exponents of the couplings measure the speed of time variation of the couplings in terms of a physical time. In fact we have checked that they are invariant under an infinitesimal change of gauge.

The results obtained in this paper are the one-loop effects. Since the IR effects at each loop level manifest as polynomials in $\log a(\tau)$, perturbation theories are broken after enough time passed. In other words, the obtained results describe physics at the initial stage $\kappa^2 H^2 \log a(\tau) \ll 1$. To investigate the eventual contributions to physical quantities, we need to evaluate the IR effects nonperturbatively. Although the IR effects from specific matter fields have been investigated nonperturbatively [9, 10, 16, 17, 11], the nonperturbative approach for the IR effects from gravitons is an open issue.

Since our Universe is dominated by dark energy, our results imply the couplings of the standard model become time dependent. Since the coefficients of IR logarithms are of $O(\kappa^2 H^2) \sim 10^{-120}$, it is not observable now. However it is relevant to the ultimate fate of the Universe as their effect grows linearly with cosmic time like Hawking radiation. This effect is much larger in the inflationary era as $\kappa^2 H^2$ could be as large as 10^{-7}.

Since the cosmological constant is a function of the couplings of the microscopic theory, it may acquire time dependence if the couplings evolve with time. So we need to investigate such effects to understand possible time dependence of the cosmological constant in addition to pure matter and gravity contributions separately. Let us follow the change of the couplings and cosmological constant under time evolution. We note here that the vanishing cosmological constant is a fixed point of the evolution where the couplings become constant. It seems conceivable that such a self-tuning mechanism may play an important role in the cosmological constant problem.

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A Derivation of (3.30)-(3.32) and (4.8)

Here we explain how to derive the identities (3.30)-(3.32) and (4.8). Each integrand in their left-hand sides is written as

$$\frac{1}{\Delta x^2} = \frac{1}{4}\partial^2 \log H^2\Delta x^2, \tag{A.1}$$

$$\frac{\Delta x_\alpha}{\Delta x^2} = \frac{1}{2}\partial_\alpha \log H^2\Delta x^2, \tag{A.2}$$

$$\frac{\Delta x_\alpha \Delta x_\beta}{\Delta x^2} = \frac{1}{2}\partial_\alpha(\Delta x_\beta \log H^2\Delta x^2) - \frac{1}{2}\eta_{\alpha\beta}\log H^2\Delta x^2, \tag{A.3}$$

$$\frac{\Delta x_\alpha \Delta x_\beta}{\Delta x^4} = -\frac{1}{4}\big\{\partial_\alpha\partial_\beta - \frac{1}{2}\eta_{\alpha\beta}\partial^2\big\}\log H^2\Delta x^2, \tag{A.4}$$

where we abbreviate the indexes $++$, $+-$ because the above identities work in both cases. By using them and leaving differential operators out of the integrals, the left-hand sides in (3.30)-(3.32) and (4.8) are

$$\begin{aligned}&\partial_\alpha\partial_\beta \int d^4x' \big[\frac{1}{\Delta x_{++}^2} - \frac{1}{\Delta x_{+-}^2}\big] \\ &= -\frac{1}{4}\delta_\alpha{}^0\delta_\beta{}^0\partial_0^4 \int d^4x' \big[\log H^2\Delta x_{++}^2 - \log H^2\Delta x_{+-}^2\big],\end{aligned} \tag{A.5}$$

$$\begin{aligned}&\partial_\beta\partial_\gamma\partial_\delta \int d^4x' \big[\frac{\Delta x_\alpha}{\Delta x_{++}^2} - \frac{\Delta x_\alpha}{\Delta x_{+-}^2}\big] \\ &= \frac{1}{2}\delta_\alpha{}^0\delta_\beta{}^0\delta_\gamma{}^0\delta_\delta{}^0\partial_0^4 \int d^4x' \big[\log H^2\Delta x_{++}^2 - \log H^2\Delta x_{+-}^2\big],\end{aligned} \tag{A.6}$$

$$\begin{aligned}&\partial_\gamma\partial_\delta\partial_\varepsilon\partial_\delta \int d^4x' \big[\frac{\Delta x_\alpha\Delta x_\beta}{\Delta x_{++}^2} - \frac{\Delta x_\alpha\Delta x_\beta}{\Delta x_{+-}^2}\big] \\ &= -\frac{1}{2}\delta_\alpha{}^0\delta_\beta{}^0\delta_\gamma{}^0\delta_\delta{}^0\delta_\varepsilon{}^0\delta_\eta{}^0\partial_0^5 \int d^4x' \ \Delta\tau\big[\log H^2\Delta x_{++}^2 - \log H^2\Delta x_{+-}^2\big] \\ &\quad -\frac{1}{2}\eta_{\alpha\beta}\delta_\gamma{}^0\delta_\delta{}^0\delta_\varepsilon{}^0\delta_\eta{}^0\partial_0^4 \int d^4x' \big[\log H^2\Delta x_{++}^2 - \log H^2\Delta x_{+-}^2\big],\end{aligned} \tag{A.7}$$

$$\begin{aligned}&\int d^4x' \ \frac{1}{\tau'^2}\big[\frac{\Delta x_\alpha\Delta x_\beta}{\Delta x_{++}^4} - \frac{\Delta x_\alpha\Delta x_\beta}{\Delta x_{+-}^4}\big] \\ &= -\frac{1}{4}\big\{\delta_\alpha{}^0\delta_\beta{}^0 + \frac{1}{2}\eta_{\alpha\beta}\big\}\partial_0^2 \int d^4x' \ \frac{1}{\tau'^2}\big[\log H^2\Delta x_{++}^2 - \log H^2\Delta x_{+-}^2\big].\end{aligned} \tag{A.8}$$

Note that the differential operator outside the integral is equal to the time derivative $\partial_\alpha \to \delta_\alpha{}^0\partial_0$. Furthermore, we have replaced Δx_α to $-\delta_\alpha{}^0\Delta\tau$ in (A.7).

As a concrete example, we calculate the following integral

$$\partial_0^4 \int d^4x' \big[\log H^2\Delta x_{++}^2 - \log H^2\Delta x_{+-}^2\big]. \tag{A.9}$$

From (3.24), the logarithm with each index $++,\ +-$ is

$$\begin{aligned}\log H^2\Delta x_{++}^2 &= \log H^2|\Delta\tau^2 - r^2| + i\pi\theta(\Delta\tau^2 - r^2),\\ \log H^2\Delta x_{+-}^2 &= \log H^2|\Delta\tau^2 - r^2| - i\pi\theta(\Delta\tau^2 - r^2)\big\{\theta(\Delta\tau) - \theta(-\Delta\tau)\big\},\end{aligned} \tag{A.10}$$

where $r^2 \equiv (\mathbf{x} - \mathbf{x}')^2$. By substituting (A.10), (A.9) is

$$\partial_0^4 \int_{-\frac{1}{H}}^{\tau} d\tau' \int 4\pi r^2 dr\ 2i\pi = \frac{8i\pi^2}{3}\partial_0^4 \int_{-\frac{1}{H}}^{\tau} d\tau'\ \Delta\tau^3 = 16i\pi^2. \tag{A.11}$$

In a similar way, the other integrals are evaluated as

$$\partial_0^5 \int d^4x'\ \Delta\tau\big[\log H^2\Delta x_{++}^2 - \log H^2\Delta x_{+-}^2\big] = 64i\pi^2, \tag{A.12}$$

$$\begin{aligned}\partial_0^2 \int d^4x'\ \frac{1}{\tau'^2}\big[\log H^2\Delta x_{++}^2 - \log H^2\Delta x_{+-}^2\big] &= 16i\pi^2 \int_{-\frac{1}{H}}^{\tau} d\tau'\ \frac{\Delta\tau}{\tau'^2}\\ &\simeq 16i\pi^2 \log a(\tau).\end{aligned} \tag{A.13}$$

We have extracted the IR logarithm in the last line of (A.13). By substituting (A.11)-(A.13) to (A.5)-(A.8), we obtain the desired identities

$$\partial_\alpha\partial_\beta \int d^4x' \big[\frac{1}{\Delta x_{++}^2} - \frac{1}{\Delta x_{+-}^2}\big] = -4i\pi^2\delta_\alpha{}^0\delta_\beta{}^0, \tag{A.14}$$

$$\partial_\beta\partial_\gamma\partial_\delta \int d^4x' \big[\frac{\Delta x_\alpha}{\Delta x_{++}^2} - \frac{\Delta x_\alpha}{\Delta x_{+-}^2}\big] = 8i\pi^2\delta_\alpha{}^0\delta_\beta{}^0\delta_\gamma{}^0\delta_\delta{}^0, \tag{A.15}$$

$$\begin{aligned}&\partial_\gamma\partial_\delta\partial_\varepsilon\partial_\eta \int d^4x' \big[\frac{\Delta x_\alpha\Delta x_\beta}{\Delta x_{++}^2} - \frac{\Delta x_\alpha\Delta x_\beta}{\Delta x_{+-}^2}\big]\\ &= -\,32i\pi^2\delta_\alpha{}^0\delta_\beta{}^0\delta_\gamma{}^0\delta_\delta{}^0\delta_\varepsilon{}^0\delta_\eta{}^0 - 8i\pi^2\eta_{\alpha\beta}\delta_\gamma{}^0\delta_\delta{}^0\delta_\varepsilon{}^0\delta_\eta{}^0,\end{aligned} \tag{A.16}$$

$$\int d^4x'\ \frac{1}{\tau'^2}\big[\frac{\Delta x_\mu\Delta x_\nu}{\Delta x_{++}^4} - \frac{\Delta x_\mu\Delta x_\nu}{\Delta x_{+-}^4}\big] \simeq -4i\pi^2\log a(\tau)\big\{\delta_\mu{}^0\delta_\nu{}^0 + \frac{1}{2}\eta_{\mu\nu}\big\}. \tag{A.17}$$

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